# Appendix

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**Abstract**

This is the Appendix for the paper "Modeling Threats and Promises: Explaining the Munich Crisis of 1938."

## A1 Terminology and Notation

This Appendix contains the analysis of the special case of the Carrot and Stick Game with incomplete information called the Munich Model. Refer to the text for discussion of the two players, Manipulator (*Man*) and Target (*Tar*). Only two types of *Man*—*PD* and *HR*—and only two types of *Tar*—*DI* and *CS*—are considered here. For convenience, we write that *Man*'s type is a member of $Y_M = \{PD, HR\}$ and *Tar*'s type is a member of $Y_T = \{DI, CS\}$. To indicate a player's type, we use notation such as *Man: PD* to say that *Man* is of type *PD*. Similarly, we define *Man: HR*, *Tar: DI*, and *Tar: CS*. *Man*'s prior (initial) type probabilities are $p_{PD}$ and $p_{HR} = 1 - p_{PD}$. Similarly, *Tar*'s type probabilities are $p_{DI}$ and $p_{CS} = 1 - p_{DI}$. There is uncertainty, as all prior type probabilities are assumed to lie strictly between 0 and 1.

The five possible outcomes are *MW, BW, SQ, C*, and *TW*. A player's utility for an outcome is determined by its type. A player's pecific utility values are chosen from the following common-knowledge utilities:

$$\begin{array}{rl} Man: & m^+_{BW} > m_{MW} > m^-_{BW} > m_{SQ} > m^+_C > m_{TW} > m^-_C \\ Tar: & t_{SQ} > t^+_{BW} > t_{TW} > t^-_{BW} > t^+_C > t_{MW} > t^-_C \end{array}$$

For example, the outcome $MW$ is always worth $m_{MW}$ to *Man*, but the outcome $BW$ may be worth either $m^+_{BW}$ or $m^-_{BW}$, depending on *Man*'s type. The relation of a player's type to its utilities is shown in Table A1. For example, the top line of the table indicates that, if *Man: PD*, then *Man*'s utility for outcome $BW$ is $m^-_{BW}$ and its utility for outcome $C$ is

$m_C^+$. Note that *Man*'s utilities for outcomes *MW*, *SQ*, and *TW* are always $m_{MW}$, $m_{SQ}$, and $m_{TW}$, regardless of *Man*'s type. On the other hand, if *Man: HR*, then *Man*'s utilities for outcomes *BW* and *C* become $m_{BW}^+$ and $m_C^-$, respectively. The situation is similar for *Tar*, whose utilities for *BW* and *C* change according to its type.

| Player | Type | Probability | Preference |
|---|---|---|---|
| *Man* | *PD* | $p_{PD}$ | $m_{MW} > m_{BW}^- > m_{SQ} > m_C^+ > m_{TW}$ |
| *Man* | *HR* | $p_{HR} = 1 - p_{PD}$ | $m_{BW}^+ > m_{MW} > m_{SQ} > m_{TW} > m_C^-$ |
| *Tar* | *DI* | $p_{DI}$ | $t_{SQ} > t_{TW} > t_{BW}^- > t_{MW} > t_C^-$ |
| *Tar* | *CS* | $p_{CS} = 1 - p_{DI}$ | $t_{SQ} > t_{BW}^+ > t_{TW} > t_C^+ > t_{MW}$ |

Table A1: Munich Model: Types, type probabilities, and preferences.

It is convenient to associate simple random variables $M_{BW}, M_C, T_{BW},$ and $T_C$ with player types, as follows:

- $Pr\{Man\text{: }PD\} = p_{PD}$. If *Man: PD*, then $M_{BW} = m_{BW}^-$ and $M_C = m_C^+$.
- $Pr\{Man\text{: }HR\} = p_{HR} = 1 - p_{PD}$. If *Man: HR*, then $M_{BW} = m_{BW}^+$ and $M_C = m_C^-$.
- $Pr\{Tar\text{: }DI\} = p_{DI}$. If *Tar: DI*, then $T_{BW} = t_{BW}^-$ and $T_C = t_C^-$.
- $Pr\{Tar\text{: }CS\} = p_{CS} = 1 - p_{DI}$. If *Tar: CS*, then $T_{BW} = t_{BW}^+$ and $T_C = t_C^+$.

The players's types are independent. The parameters of the game are the players' utilities and type probabilities. We assume that parameters are never equal, that they never equal 0 or 1, and that any functional relationships among the parameters occur only on sets of measure zero, which we ignore.

Referring to Figure 1 of the text, *Man*'s type-dependent strategies relate to *Man*'s decision at Node 1. In general, $x$ denotes the probability that *Man* chooses Demand, and $1 - x$ the probability that *Man* chooses Concede. This choice is type-dependent, so the probability that *Man: PD* chooses Demand is denoted $x_{PD}$, and the probability that *Man: HR* chooses Demand is denoted $x_{HR}$.

Should the game reach Node 2, *Tar* adjusts *Man*'s type probabilities from their prior values, to their posterior values, reflecting that *Man* has been observed to choose Demand and not Concede. The adjusted (posterior) probabilities are denoted $q_{PD}$ and $q_{HR}$. (See (2) below.) Note that $q_{HR} = 1 - q_{PD}$.

At Node 2, *Tar* chooses either Resist, with probability $y$, or Comply, with probability $1 - y$. Again, these choices are type-dependent; the probability that *Tar: DI* chooses Resist is denoted $y_{DI}$, and the probability that *Tar: CS* chooses Resist is denoted $y_{CS}$.

Because the game terminates immediately after *Man*'s choice at Node 3 or Node 4, those choices depend only on *Man*'s type. At Node 3, *Man: PD* chooses Renege and *Man: HR* chooses Honor. At Node 4, *Man: PD* chooses Press On and *Man: HR* chooses Back Down.

## A2 Perfect Bayesian Equilibria

A Perfect Bayesian Equilibrium (PBE) consists of a 5-tuple of probabilities,

$$(x; y; q) = (x_{PD}, x_{HR}; y_{DI}, y_{CS}; q_{PD}), \tag{1}$$

in which the type probability, $q_{PD}$, is updated using Bayesian principles, and all strategic variables are chosen to maximize expected utility (calculated according to appropriately updated probabilities).

At a Perfect Bayesian Equilibrium, if the game reaches Node 2, *Tar* updates *Man*'s type probabilities $p_{PD}$ and $p_{HR} = 1 - p_{PD}$ to

$$q_{PD} = \frac{p_{PD}x_{PD}}{p_{PD}x_{PD} + p_{HR}x_{HR}} \tag{2}$$

and, of course, $q_{HR} = 1 - q_{PD}$. Note that the denominator on the right side of (2), $p_{PD}x_{PD} + p_{HR}x_{HR}$, is equal to 0 if and only if $x_{PD} = x_{HR} = 0$; in this case, the game can never reach Node 2, so the updating specified in (2) does not apply. In other words, (2) restricts a PBE if and only if either $x_{PD} > 0$ or $x_{HR} > 0$. If so, then $q_{PD}$ is well-defined, and satisfies $0 \leq q_{PD} \leq 1$.

To study *Tar*'s choice at Node 2, we first note that, should the game reach Node 3, the outcome is always *MW* if *Man: PD* and *BW* if *Man: HR*. Similarly, should the game reach Node 4, the outcome is always *C* if *Man: PD* and *TW* if *Man: HR*. Of course, *Tar* does not know *Man*'s true type, and must base its decision at Node 2 on its updated type probabilities, $q_{PD}$ and $q_{HR} = 1 - q_{PD}$.

For $j \in Y_T$, suppose that *Tar:j*. Then *Tar*'s expected utility at Node 2 can be written

$$EU_{T|j} = y_j \left[q_{PD}T_C + (1 - q_{PD})t_{TW}\right] + (1 - y_j)\left[q_{PD}t_{MW} + (1 - q_{PD})T_{BW}\right]$$

where the values of $T_C$ and $T_{BW}$ are determined by *Tar*'s type, $j$, as specified above. Differentiating $EU_{T|j}$ with respect to $y_j$ produces

$$\begin{aligned}\frac{\partial EU_{T|j}}{\partial y_j} &= q_{PD}T_C + (1 - q_{PD})t_{TW} - q_{PD}t_{MW} - (1 - q_{PD})T_{BW} \\ &= q_{PD}(T_C - t_{MW}) + (1 - q_{PD})(t_{TW} - T_{BW}) \equiv H_j. \end{aligned} \tag{3}$$

At any PBE, $y_j = 0$ if $H_j < 0$ and $y_j = 1$ if $H_j > 0$.

Suppose that $j = DI$, i.e., that *Tar: DI*. Then $T_C - t_{MW} = t_C^- - t_{MW} < 0$ and $t_{TW} - T_{BW} = t_{TW} - t_{BW}^- > 0$, so that

$$H_{DI} = q_{PD}(t_C^- - t_{MW}) + (1 - q_{PD})(t_{TW} - t_{BW}^-),$$

and it follows that $H_{DI} > 0$ if and only if

$$q_{PD} < n_{DI} = \frac{t_{TW} - t_{BW}^-}{t_{MW} - t_C^- + t_{TW} - t_{BW}^-}.$$

Therefore, if $q_{PD} < n_{DI}$, then $y_{DI} = 1$ at PBE ; if $q_{PD} > n_{DI}$, then $y_{DI} = 0$ at PBE . Note that $0 < n_{DI} < 1$.

Finally, suppose that *Tar: CS*. Then $T_C = t_C^+$ and $T_{BW} = t_{BW}^+$, so that

$$H_{CS} = q_{PD}(t_C^+ - t_{MW}) + (1 - q_{PD})(t_{TW} - t_{BW}^+)$$

and it follows that $H_{CS} > 0$ if and only if

$$q_{PD} > n_{CS} = \frac{t_{BW}^+ - t_{TW}}{t_C^+ - t_{MW} + t_{BW}^+ - t_{TW}}.$$

Therefore, if $q_{PD} > n_{CS}$, then $y_{CS} = 1$ at PBE; if $q_{PD} < n_{CS}$, then $y_{CS} = 0$ at PBE. Again, $0 < n_{CS} < 1$.

Thus, if $q_{PD}$ is close enough to 0, $y_{DI} = 1$ and $y_{CS} = 0$, whereas if $q_{PD}$ is close enough to 1, $y_{DI} = 0$ and $y_{CS} = 1$. For middling values of $q_{PD}$, both $y_{DI}$ and $y_{CS}$ equal 0 at equilibrium if $n_{DI} < n_{CS}$, while both $y_{DI}$ and $y_{CS}$ equal 1 at equilibrium if $n_{DI} > n_{CS}$. Whether $n_{DI}$ is greater than or less than $n_{CS}$ depends on the specific values of *Tar*'s utilities. (As noted earlier, we neglect the possibility of equality.)

Now consider *Man*'s choice at Node 1, and recall that *Man* is of type *PD* or *HR*. *Man*'s strategic variables, $x_{PD}$ and $x_{HR}$, are the probabilities that *Man* chooses Demand rather than Concede if *Man: PD* and *Man: HR*, respectively.

Suppose for the moment that *Man: PD* and *Tar: DI*. Then the outcome will be *MW* if *Tar* chooses Comply (probability $1 - y_{DI}$) and *C* if *Tar* chooses Resist (probability $y_{DI}$). Thus, *Man*'s utility will be $m_{MW}$ if *Tar* chooses Comply at Node 2, and $m_C^+$ if *Tar* chooses Resist at Node 2. It follows that, if *Man: PD* and *Tar: DI*, then *Man*'s expected utility if *Man* chooses Demand is

$$(1 - y_{DI})m_{MW} + y_{DI}m_C^+$$

It follows that, if *Man:PD* and *Tar: DI*, *Man*'s expected utility at Node 1 is

$$x_{PD}\{(1 - y_{DI})m_{MW} + y_{DI}m_C^+\} + (1 - x_{PD})m_{SQ}$$

Now, continue to assume that *Man: PD* but consider both types of *Tar*. By similar reasoning, *Man*'s expected utility at Node 1 is

$$\begin{aligned}
EU_{M|PD} = x_{PD}\{&p_{DI}[(1-y_{DI})m_{MW} + y_{DI}m_C^+] \\
&+p_{CS}[(1-y_{CS})m_{MW} + y_{CS}m_C^+]\} + (1-x_{PD})m_{SQ} \\
&= x_{PD}\left\{m_{MW}\sum_{j\in Y_T} p_j(1-y_j) + m_C^+\sum_{j\in Y_T} p_j y_j\right\} + (1-x_{PD})m_{SQ}
\end{aligned}$$

Now observe that

$$\sum_{j\in Y_T} p_j(1-y_j) + \sum_{j\in Y_T} p_j y_j = \sum_{j\in Y_T} p_j[(1-y_j)+y_j] = \sum_{j\in Y_T} p_j = 1$$

Therefore, writing $r(p,y) = \sum_{j\in Y_T} p_j y_j = p_{DI}y_{DI} + (1-p_{DI})y_{CS}$, we have that

$$\begin{aligned}
EU_{M|PD} &= x_{PD}\{m_{MW}(1-r(p,y)) + m_C^+ r(p,y)\} + (1-x_{PD})m_{SQ} \\
&= m_{SQ} + x_{PD}\left[(m_C^+ - m_{SQ})r(p,y) + (m_{MW}-m_{SQ})(1-r(p,y))\right]
\end{aligned}$$

Differentiating with respect to $x_{PD}$ yields

$$\frac{\partial EU_{M|PD}}{\partial x_{PD}} = (m_{MW} - m_{SQ}) + (m_C^+ - m_{MW})r(p,y) \equiv J_{PD}.$$

Observe that $J_{PD}$ is an indicator of the value of $x_{PD}$ at a PBE. Now $J_{PD} \geq 0$ if and only if

$$r(p,y) \leq u_{PD} = \frac{m_{MW} - m_{SQ}}{m_{MW} - m_C^+}$$

so it follows that, if $r(p,y) < u_{PD}$, then $x_{PD} = 1$ at PBE, and, if $r(p,y) > u_{PD}$, then $x_{PD} = 0$ at PBE. Note that $0 < u_{PD} < 1$.

By similar reasoning,

$$EU_{M|HR} = m_{SQ} + x_{HR}\left[(m_{TW} - m_{SQ})r(p,y) + (m_{BW}^+ - m_{SQ})(1-r(p,y))\right]$$

so that $J_{HR}$, the indicator of the value of $x_{HR}$ at a PBE, is defined by

$$\frac{\partial EU_{M|HR}}{\partial x_{HR}} = (m_{BW}^+ - m_{SQ}) + (m_{TW} - m_{BW}^+)r(p,y) \equiv J_{HR}$$

Now $J_{HR} \geq 0$ if and only if

$$r(p,y) \leq u_{HR} = \frac{m_{BW}^+ - m_{SQ}}{m_{BW}^+ - m_{TW}}$$

so it follows that, if $r(p,y) < u_{HR}$, then $x_{HR} = 1$ at PBE, and, if $r(p,y) > u_{HR}$, then $x_{HR} = 0$ at PBE. Again, $0 < u_{HR} < 1$. Also, $u_{HR}$ may be greater than or less than $u_{PD}$. (As usual, we ignore the possibility that $u_{HR} = u_{PD}$.)

To summarize, $r(p, y)$, which equals the unconditional probability that *Tar* chooses Resist, determines *Man*'s strategic choice at equilibrium. If $r(p, y)$ is near 0, both types of *Man* choose $x = 1$. If $r(p, y)$ is near 1, both types of *Man* choose $x = 0$. But the two types of *Man* have different thresholds. *Man: PD* can choose $x_{PD} = 1$ at PBE only if $r(p, y) \leq u_{PD}$, while *Man: HR* can choose $x_{HR} = 1$ at PBE only if $r(p, y) \leq u_{HR}$. These observations allow for a middling zone of values of $r(p, y)$ where one of $x_{PD}$ and $x_{HR}$ equals 1, and the other equals 0, at a PBE.

In summary, a Perfect Bayesian Equilibrium (PBE) consists of a 5-tuple of probabilities,

$$(x; y; q) = (x_{PD}, x_{HR}; y_{DI}, y_{CS}; q_{PD}),$$

such that

**(A)** If $x_{PD} > 0$ or $x_{HR} > 0$, then

$$q_{PD} = \frac{p_{PD} x_{PD}}{p_{PD} x_{PD} + (1 - p_{PD}) x_{HR}}$$

**(B)**

$$y_{DI} = \begin{cases} 1 & \text{if } q_{PD} < n_{DI} \\ 0 & \text{if } q_{PD} > n_{DI} \end{cases}; \quad y_{CS} = \begin{cases} 0 & \text{if } q_{PD} < n_{CS} \\ 1 & \text{if } q_{PD} > n_{CS} \end{cases}$$

**(C)** If $r = p_{DI} y_{DI} + (1 - p_{DI}) y_{CS}$, then

$$x_{PD} = \begin{cases} 1 & \text{if } r < u_{PD} \\ 0 & \text{if } r > u_{PD} \end{cases}; \quad x_{HR} = \begin{cases} 1 & \text{if } r < u_{HR} \\ 0 & \text{if } r > u_{HR} \end{cases}$$

# A3 Munich Model: Perfect Bayesian Equilibria

The parameter pairs $n_{DI}$ and $n_{CS}$, and $u_{HR}$ and $u_{PD}$, shape the PBE of the Carrot and Stick Game, as can be seen by examining **B** and **C**. In the text, it is argued that the most likely configuration of these parameters in the model under study is $n_{CS} < n_{DI}$ and $u_{PD} < u_{HR} < 1 - u_{PD}$. (The latter inequality implies that $u_{PD} < \frac{1}{2}$.)

**Lemma 1**: If $(x_{PD}, x_{HR}; y_{DI}, y_{CS}; q_{PD})$ is a PBE, then

(a) Whenever $x_{PD} > 0, x_{HR} = 1$;

(b) Whenever $y_{DI} < 1, y_{CS} = 1$.

**Proof**: To prove (a), note that it follows from **(C)** that, if $x_{PD} > 0$ at PBE, then $r \leq u_{PD}$. Because $u_{PD} < u_{HR}$, we have that $r < u_{HR}$. Now **(C)** shows that $x_{HR} = 1$, proving (a). To prove (b), use **(B)** to show that, if $y_{DI} < 1$, it must be the case that $q_{PD} \geq n_{DI} > n_{CS}$, which by **(B)** shows that $y_{CS} = 1$. The proof of (d) is similar. ■

Note that Lemma 1(a) depends only on the relation of $u_{PD}$ and $u_{HR}$, and Lemma 1(b) depends only on the relation of $n_{CS}$ and $n_{DI}$.

Remarkably, Lemma 1 permits classification of the possible PBE of the game according to the values of $x_{PD}$ and $x_{HR}$. The left-hand column of Table A2 indicates all possible PBE of the Carrot and Stick Game with the configuration of parameters detailed above.

| Non-Deterrence | | Deterrence | | |
|---|---|---|---|---|
| E1 | $(x_{PD} = 1, x_{HR} = 1)$ | ED1 | $q_{PD} < n_{DI}$ | $(y_{DI} = 1, y_{CS} = 0)$ |
| E2 | $(x_{PD} = 0, x_{HR} = 1)$ | ED2 | $q_{PD} = n_{CS}$ | $(y_{DI} = 1, 0 < y_{CS} < 1)$ |
| E3 | $(0 < x_{PD} < 1, x_{HR} = 1)$ | ED3 | $n_{CS} < q_{PD} < n_{DI}$ | $(y_{DI} = 1, y_{CS} = 1)$ |
| E4 | $(x_{PD} = 0, 0 < x_{HR} < 1)$ | ED4 | $q_{PD} = n_{DI}$ | $(0 < y_{DI} < 1, y_{CS} = 1)$ |
| | | ED5 | $q_{PD} > n_{DI}$ | $(y_{DI} = 0, y_{CS} = 1)$ |

Table A2: Munich Model: Non-Deterrence and Deterrence Equilibria

One PBE is missing from the left half of Figure A2, namely the Deterrence PBE, which could be entered as ED $(x_{PD} = 0, x_{HR} = 0)$. At all Deterrence PBE, *Man* never Demands, so Node 2 is off the equilibrium path, and the outcome is always *SQ*. Note that **(A)** does not apply, so the value of $q_{PD}$ is formally unspecified. Nonetheless, $q_{PD}$ must have a value at a PBE, and its value must determine $y_{DI}$ and $y_{CS}$ according to **(B)**, and those two values in turn determine $r$. Moreover, by **(C)** we must have $r = p_{DI}y_{DI} + (1 - p_{DI})y_{CS} \geq \max\{u_{PD}, u_{HR}\}$.

The right half of Table A2 breaks down all of the Deterrence PBE that are in fact Sequential (Kreps and Wilson, 1982). At a Sequential Deterrence Equilibrium, care must be taken to establish the value of $q_{PD}$, the belief at Node 2, as that node never arises in play. This value must be the limit of beliefs, calculated by **(A)**, for some sequence of strategies $(x_{PD}, x_{HR})$ such that $x_{PD} > 0$, $x_{HR} > 0$, and $\lim(x_{PD}, x_{HR}) = (0, 0)$. For example, for $\epsilon > 0$, let $x_{PD} = \epsilon$ and $x_{HR} = \epsilon^2$, so that $\lim_{\epsilon \to 0+}(x_{PD}, x_{HR}) = (0, 0)$. By **(A)**,

$$q_{PD} = \frac{\epsilon p_{PD}}{\epsilon p_{PD} + \epsilon^2(1 - p_{PD})} = \frac{1}{1 + \epsilon \frac{1 - p_{PD}}{p_{PD}}}$$

so that $\lim_{\epsilon \to 0+} q_{PD} = 1$ because $0 < p_{PD} < 1$. Thus $q_{PD} = 1$ is a possibility at a Sequential Deterrence Equilibrium. So is $q_{PD} = 0$, as can be seen by reversing the assignment of $\epsilon$ and $\epsilon^2$. Finally, for any $q$ such that $0 < q < 1$, set $x_{PD} = \epsilon$ and $x_{HR} = \epsilon \frac{1-q}{q} \frac{p_{PD}}{1-p_{PD}}$. Then it is easy to verify that $\lim_{\epsilon \to 0+} q_{PD} = q$. We conclude that any value of $q_{PD} \in [0, 1]$ is possible at a Sequential Deterrence Equilibrium. Using this fact, and **(B)**, we can classify all Sequential Deterrence equilibrium according to the value of $q_{PD}$, as shown in the right half of Table A2.

Before identifying the Sequential Deterrence equilibria, we study the non-deterrence PBE, which appear in the left half of Table A2. Recall that we are assuming that $u_{PD} < u_{HR} < 1 - u_{PD}$ and $n_{CS} < n_{DI}$.

**PBE of class E1:** $(x_{PD}, x_{HR}) = (1, 1)$. We search first for pure-strategy PBE in which both types of *Man* Demand for certain. By **(A)**, $q_{PD} = p_{PD}$ at any such PBE. Because of **(C)** and $u_{PD} < u_{HR}$, it must be the case that $r = p_{DI}y_{DI} + (1 - p_{DI})y_{CS} \leq u_{PD}$. We can assume that *Tar*'s strategy must be pure when $x_{PD} = x_{HR} = 1$ because, by **(B)**, a mixed strategy for *Tar* requires either $p_{PD} = n_{DI}$ or $p_{PD} = n_{CS}$; these conditions are equalities in parameter space, which we ignore. The possibility that $(y_{DI}, y_{CS}) = (1, 1)$ must also be rejected, because it implies that $r = 1 > u_{PD}$. The remaining possibilities are $(y_{DI}, y_{CS}) = (1, 0)$ or $(0, 1)$ or $(0, 0)$. But it follows from Lemma 1(b) that $(y_{DI}, y_{CS}) = (0, 0)$ cannot occur at a PBE. Therefore, a PBE with $(x_{PD}, x_{HR}) = (1, 1)$ must satisfy either (a) $(y_{DI}, y_{CS}) = (1, 0)$, or (b) $(y_{DI}, y_{CS}) = (0, 1)$.

In case (a), we must have $q_{PD} \leq n_{DI}$ and $q_{PD} \leq n_{CS}$, where $q_{PD} = p_{PD}$. Because $n_{DI} > n_{CS}$, both conditions hold if and only if $p_{PD} \leq n_{CS}$. But now $r = p_{DI}$, so from **(C)** we must have $p_{DI} \leq u_{PD}$. In summary, we have found the PBE

> **E1a**: $(x_{PD}, x_{HR}; y_{DI}, y_{CS}; q_{PD}) = (1, 1; 1, 0; p_{PD})$. **E1a** exists if and only if $0 \leq p_{PD} \leq n_{CS}$ and $0 \leq p_{DI} \leq u_{PD}$.

In case (b), where $(y_{DI}, y_{CS}) = (0, 1)$, we must have $p_{PD} = q_{PD} \geq n_{DI}$ and $r = 1 - p_{DI} \leq u_{PD}$, or $p_{DI} \geq 1 - u_{PD}$. This leads us to the PBE

> **E1b**: $(x_{PD}, x_{HR}; y_{DI}, y_{CS}; q_{PD}) = (1, 1; 0, 1; p_{PD})$. **E1b** exists if and only if $n_{DI} \leq p_{PD} \leq 1$ and $1 - u_{PD} \leq p_{DI} \leq 1$.

**PBE of class E2:** $(x_{PD}, x_{HR}) = (0, 1)$. Now we search for PBE at which *Man: HR* always Demands and *Man: PD* never Demands. At such a PBE, $q_{PD} = 0$, by **(A)**. By **(B)**, $y_{DI} = 1$ and $y_{CS} = 0$. Therefore $r = p_{DI}$. By **(C)**, $u_{PD} < S < u_{HR}$. There is only one possibility for this PBE,

> **E2**: $(x_{PD}, x_{HR}; y_{DI}, y_{CS}; q_{PD}) = (0, 1; 1, 0; 0)$. **E2** exists if and only if $u_{PD} \leq p_{DI} \leq u_{HR}$.

Note that the existence of **E2** does not depend on the value of $p_{PD}$.

**PBE of class E3:** $0 < x_{PD} < 1, x_{HR} = 1$. Now we search for PBE at which *Man: HR* Demands for certain while *Man: PD* sometimes Demands and sometimes does not. By **(A)**,

$$q_{PD} = \frac{p_{PD}x_{PD}}{p_{PD}x_{PD} + (1 - p_{PD})}.$$

By **(C)**, $r = p_{DI}y_{DI} + (1 - p_{DI})y_{CS} = u_{PD}$, which as usual shows that *Tar*'s strategy must be mixed. By Lemma 1(b), there are two possibilities: either (a) $y_{DI} = 1$ and $0 < y_{CS} < 1$, or (b) $0 < y_{DI} < 1$ and $y_{CS} = 1$.

Assume (a), so that $r = p_{DI} + y_{CS}(1 - p_{DI}) = u_{PD}$, which is equivalent to

$$y_{CS} = \frac{u_{PD} - p_{DI}}{1 - p_{DI}} \equiv y^a_{CS}.$$

Note that $0 < y^a_{CS} < 1$ if and only if $0 \leq p_{DI} < u_{PD}$. By **(B)**, $q_{PD} = n_{CS}$, which is equivalent to

$$x_{PD} = \frac{(1 - p_{PD})n_{CS}}{p_{PD}(1 - n_{CS})} \equiv x^a_{PD}.$$

Notice that $0 < x^a_{PD} < 1$ if and only if $n_{CS} < p_{PD} < 1$. The PBE we have found is

> **E3a**: $(x_{PD}, x_{HR}; y_{DI}, y_{CS}; q_{PD}) = (x^a_{PD}, 1; 1, y^a_{CS}; n_{CS})$. **E3a** exists if and only if $n_{CS} < p_{PD} < 1$ and $0 \leq p_{DI} < u_{PD}$.

Now assume possibility (b), $0 < y_{DI} < 1$ and $y_{CS} = 1$. Then by **(C)**, $r = y_{DI}p_{DI} + (1 - p_{DI}) = u_{PD}$, which is equivalent to

$$y_{DI} = \frac{u_{PD} - 1 + p_{DI}}{p_{DI}} \equiv y^b_{DI}.$$

Note that $0 < y^b_{DI} < 1$ if and only if $1 - u_{PD} < p_{DI} \leq 1$. At this PBE, we must have $q_{PD} = n_{DI}$ by **(B)**. This equality is equivalent to

$$x_{PD} = \frac{(1 - p_{PD})n_{DI}}{p_{PD}(1 - n_{DI})} \equiv x^b_{PD}.$$

Notice that $0 < x^b_{PD} < 1$ if and only if $n_{DI} < p_{PD} < 1$. We have found the PBE

> **E3b**: $(x_{PD}, x_{HR}; y_{DI}, y_{CS}; q_{PD}) = (x^b_{PD}, 1; y^b_{DI}, 1; n_{DI})$. **E3b** exists if and only if $n_{DI} < p_{PD} < 1$ and $1 - u_{PD} < p_{DI} \leq 1$.

**PBE of class E4:** $x_{PD} = 0, 0 < x_{HR} < 1$. Next we search for PBE at which *Man: HR* sometimes Demands while *Man: PD* never Demands. By **(A)**, $q_{PD} = 0$, so $y_{DI} = 1$ and $y_{CS} = 0$, and therefore $S = p_{DI}$. But, by **(C)**, $r = u_{HR}$. Thus an equilibrium of class **E4** can exist only when $p_{DI} = u_{HR}$, which is an equality in parameter space, so we ignore it.

We now move to the Sequential Deterrence equilibria, which are described in the right half of Table A2. It is straightforward to verify that

**ED1**: $(x_{PD}, x_{HR}; y_{DI}, y_{CS}; q_{PD}) = (0, 0; 1, 0; q_{PD})$. **ED1** exists if and only if $u_{PD} \leq p_{DI} \leq 1$. If so, **ED1** occurs whenever $0 \leq q_{PD} < n_{CS}$.

**ED2**: $(x_{PD}, x_{HR}; y_{DI}, y_{CS}; q_{PD}) = (0, 0; 1, y_{CS}; n_{CS})$. **ED2** exists if and only if $0 \leq p_{DI} < u_{PD}$. If so, **ED2** occurs whenever $y^a_{CS} \leq y_{CS} \leq 1$.

**ED3**: $(x_{PD}, x_{HR}; y_{DI}, y_{CS}; q_{PD}) = (0, 0; 1, 1; q_{PD})$. **ED3** exists for all values of $p_{PD}$ and $p_{DI}$. **ED3** occurs whenever $q_{PD}$ satisfies $n_{CS} < q_{PD} < n_{DI}$.

**ED4**: $(x_{PD}, x_{HR}; y_{DI}, y_{CS}; q_{PD}) = (0, 0; y_{DI}, 1; n_{DI})$. **ED4** exists if and only if $1 - u_{PD} < p_{DI} \leq 1$. If so, **ED4** occurs whenever $y^b_{DI} \leq y_{DI} \leq 1$.

**ED5**: $(x_{PD}, x_{HR}; y_{DI}, y_{CS}; q_{PD}) = (0, 0; 0, 1; q_{PD})$. **ED5** exists if and only if $0 < p_{DI} \leq 1 - u_{PD}$. If so, **ED5** occurs whenever $q_{PD} > n_{DI}$.

Note that the existence of any Deterrence equilibrium never depends on the value of $p_{PD}$.

In summary, at every point of the $(p_{PD}, p_{DI})$ unit square there are three families of Deterrence Equilibria: **ED1** above $p_{DI} = u_{PD}$ and **ED2** below it; **ED4** above $p_{DI} = 1 - u_{PD}$ and **ED5** below it; and **ED3** everywhere. The non-deterrence equilibria are more scattered. **E1a** and **E3a** always lie adjacent to the $p_{PD}$ axis, and never overlap, **E2** arises in a central band parallel to the $p_{PD}$ axis, and **E1b** and **E3b** occur together in a rectangle containing the point $(p_{PD}, p_{DI}) = (1, 1)$, except that **E1b** exists on all boundaries of that rectangle, while **E3b** does not. Note that there is a large region where there are no equilibria but deterrence.

Figure 2 in the text shows the equilibria of the Munich Model: (a) All Deterrence Equilibria—in three parts, because of the overlaps; (b) All Non-Deterrence Equilibria.